"TO AND FRO" BLOOD FLOW MATHEMATICAL MODEL IN VENOUS FLAP SURVIVAL

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ABSTRACT
The aim of the research is to determine "to and fro" venous flap blood circulation perfusion ability and flap size survival.

Materials and methods: "To and fro" blood circulation mathematical model based on the theory of compressed liquid flow through the porous medium.

Result: Designed mathematical model allows to calculate the viable flap size that depends on pedicle vessel radius, the blood pressure gradient, the blood viscosity, the elastic tissue capacity, the pulse frequency, vessels flap topography and surgical pedicle flap technique. Viable flap width may extent up to 4-6 cm.

Conclusions: Only thin skin or fascial flaps, were through venous pedicle with a lot of side branches located under the flap surface and along central flap axis may survive with "to and fro" blood circulation.

KEY WORDS: venous flap, blood circulation, mathematical model

INTRODUCTION
Arterialized venous free flap transplantation is popular in the area of plastic surgery for covering the skin defects [1-5]. However, mechanisms of the survival of the flap is still unclear and the necrosis of the soft tissue after transplantsations flap is still possible [6,7].

For the explanation of the tissue survival Thatte M.R. at all (1989) offered the mathematical model of pendulum blood venous supply of the flap [8]. According to this model the physiological blood circulation within flap vessels is absent. Instead it, blood moves through the central venous flap’s pedicle and its terminal branches according the “to and fro” principle. During systolic high pressure the portion of “fresh” blood passes though the flap vessels. And diastolic low pressure pushes blood out from the flap’s vessels. Authors supposed that flap vessels net can be compared with the ideal mix reactor, where “work off” blood is blended in a moment with “fresh” portion. As the result, according to this model the flap and its pedicle may have any size for survival. But experimental and clinical researches demonstrated about 5% flap’s necrosis still occurs.

THE AIM
The aim of the research was to identify the efficiency of the “to and fro” venous flap’s blood circulation perfusion and flap’s size for survival.

MATERIALS AND METHODS
Mathematical model. Takes to account, that flapе with λ thickness has radius R of the vascular pedicle (fig.1). Sinusoidal blood pressure flap inflow has maximal height P₀ (difference between systolic and diastolic pressure) and duration time T that depends on pulse frequency. The flap tissue compare with entire porous medium that consists of the pore middle radius r and has ε porosity (vascular flap net volume to entire flap volume ratio).

The pressure gradient moves blood along axial vessel with Uₓ(y,t) speed. Then blood enters into a flap’s tissue from branches side, and reaches speed component along axis X: Uₓ(x,y,t). The main idea of the task is to determine axial and side vessels branches blood moves with speed in any time. After stated calculations there is a possibility to determine “fresh” blood boundary transferred from initial time moment at flap entrance point (x=0, y=0). Maximal “fresh” blood bound flap tissue transfers was corresponded to viabil flap size.

RESULTS
MATHEMATICAL MODEL FORMULATION
Because of vessels wall elasticity, the flap blood circulation measuring is equivalent to compressed liquid flow task. Compressed liquid flow differential equation in porous
medium for one-dimensional case was calculated by the following formula [9]:

\[ \frac{\varepsilon}{\rho} \frac{\partial p}{\partial t} = - \frac{\partial (\rho U_x)}{\partial x}, \]  

where \( \rho \) – blood density; \( \varepsilon \) – flap tissue porosity; \( U_x \) – blood move speed.

Liquid speed move, according to Puazeil low [10], is proportional to pressure gradient that was calculated by the following formula [10]:

\[ U_x = -\frac{r^2}{8\mu} \frac{\partial p}{\partial x}, \]  

where \( \mu \) – tenacity.

Take in account that liquid density in equation I is identical with a liquid quantity in volume flap unity, – the dependence low of this quantity from pressure was calculated by the following formula:

\[ \rho = \rho_0 e^{g \beta P}, \]  

where coefficient \( \beta \) have a value of relative blood tissue volume change from unit pressure change.

After the mathematical transformations receive differential equation for pressure change in dimensionless quantities was calculated by the following formula [IV]:

\[ \frac{\partial P(X, \tau)}{\partial \tau} = \frac{b \partial^2 P(X, \tau)}{\partial X^2}; \]  

where dimensionless parameters are:

\[ X = \frac{x}{\lambda}; \tau = \frac{t}{T}; P = \frac{p}{p_m}; b = \frac{r^2T}{8\mu \beta^2} \]

Initial conditions for equation IV \( \tau=0; P(x,0)=0 \) and boundary condition \( \tau \geq 0; x=0; P(0,\tau)=P(Y) \), where \( P(Y) \) is a solution of equation for blood pressure changes in vessel pedicle. This equation was deduced on the basis that blood part force out from axial vessel to flap side vessels branches. With this, speed blood displace to flap is proportional to pressure gradient axis X along in axis Y every point (with \( X=0 \}). Finally, variables pressure change law in central vessel was calculated by the following formula [V]:

\[ \frac{\partial P(Y, \tau)}{\partial \tau} = a \frac{\partial^2 P(Y, \tau)}{\partial Y^2} + \sigma a \frac{\partial P(Y, \tau)}{\partial X} \bigg|_{X=0}; \]  

where \( Y = \frac{Y}{R} \) – dimensionless coordinate;

\[ a = \frac{T}{8\beta}; \sigma = \frac{2e}{\pi R^2} \]

Under condition \( Y=0; P=1 \). Differential equation system IV, V with corresponding limited condition completely describe flap blood move task. Integration of this equations determine speed blood field in flap vessels.

**MATHEMATICAL MODEL DECISION**

Equations IV, V direct integration very complicated. Some simplify is needed. Ignore wall vessels pedicle pressure oscillations. In this case equation V left part will be equal zero and it decision with corresponding boundary conditions was calculated by the following formula [VI]:

\[ P(Y) = e^{g Y}; \]  

where \( g \) constant is expressed through already known quantities according to the next formula:

\[ g^2 = \frac{\sigma}{2 \sqrt{6}} \]

As a result, in limits of made simplifications, immediately after blood pulse along vascular pedicle, pressure distributed time independently (\( t \) – till end pulse) and described by equation VI.

Use equation VI as boundary condition for equation IV. Then receive a flap pressure change low and calculated by the following formula [VII]:

\[ P(x, \tau) = e^{g Y} \text{erfc} \left( \frac{x}{2 \sqrt{b \tau}} \right); \]  

where \( \text{erfc} \ z = 1 - \text{erf} \ z = 1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt \) – additional mistakes function.
BLOOD MOVE EQUATION DECISION
AND DEFINE A SURVIVE FLAP SIZE
DEFINE FORMULA
Blood move speed along axis X and Y defines according equation II. For dimensionless quantities was calculated by the following formula:

\[ U_x = bq \frac{\partial P}{\partial x} \]  
(VIII)

where \( q = P_m P \).

Take in account VI, vascular pedicle blood transfer equation was calculated by the following formula IX:

\[ \frac{dY}{d\tau} = \gamma e^{-gY} \]  
(IX)

where \( \gamma = ag \).

As initial condition \( Y=0 \) with \( \tau=0 \), and maximal blood transfers during time \( \tau=T \), maximal flap length \( L_m \) in natural units equation will calculated by the following formula X:

\[ L_m = R^2 \left( \frac{\pi T}{2R^2v^2} \ln\left(1 + \frac{P_m e^{2Y}}{8R^2 \sqrt{\pi \mu}} \right) \right); \]

(X)

Maximal flap length in very good proximity in the case of flap thickness is not exceed 2 times vascular pedicle diameter, according to formula X, was measured:

\( L_m = 120R \)

Flap width on every vascular pedicle distance \( 0<Y<Y_m \) was calculated by integration of equation VIII and have such view calculated by the following formula XI:

\[ X(Y) = X_m e^{-gY} \left(1 - \frac{e^{gY} - 1}{gY} \right) \]  
(XI)

Finally maximal vital flap width \( X_m \) is equal to value calculated by the following formula XII:

\[ X_m = P_m e^{2\tau} \sqrt{\frac{2TP}{\pi \mu}} \]  
(XII)

According to our calculations, very thin skin or fascial flaps, were through venous pedicle with a lot of side branches is located under the flap surface and along central flap axis may survive.

As the result, the diagram (fig. 2) reflects the viable flap width blood pressure depends on factors such as: venous pressure gradient (VP) distal extremities parts maximal flap width \( X_{max} \): 1-2 cm; arterial pressure gradient (AP) maximal flap width up to 4-6 cm [5, 6, 7].

Another, biological question is about flap tissue metabolism in pendulum circulation. Perhaps, it mostly occurs out of capillary vessels, because of large erythrocyte size, bigger than capillary foramen. Physiological erythrocyte capillary inflow is possible after its transformation, that became longer and narrow inside capillary. If erythrocytes enter, under the pressure gradient is possible, – erythrocyte departures from capillary in pendulum circulation had no pressure support. Step by step, capillary vessels became blocked and capillary metabolism within the neovascularized flap was restored after some days [7, 8].

CONCLUSIONS
The mathematical model developed of “to and fro” venous blood circulation of the pedicle flap allows to calculate the viable flap size that depends on the pedicle vessel radius, blood pressure gradient, blood viscosity, elastic tissue capacity, pulse frequency, vessels flap topography and surgical pedicle flap technique. Multidirectional blood circulation of the flaps provides the survive of the thin fascial flaps with limited size about 4-6 cm.

REFERENCES
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Conflict of interest:
The Authors declare no conflict of interest.

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